

Current ElectricityPower in A.C. Circuit :-

In general the rate of doing work is called power. The power in an electrical circuit is the rate at which electrical energy is consumed in the circuit. As in an A.C. circuit both the applied e.m.f. and the current vary continuously with time, hence the power in an A.C. circuit is equal to the product of the instantaneous e.m.f. and instantaneous current averaged over a complete cycle. The power of an A.C. circuit depends on the fact whether a phase difference between the current and e.m.f. exists or not and if it exists the question arises, how much?

In an A.C. circuit the instantaneous values of the voltage and current are given by

$$E = E_0 \sin \omega t \quad \text{and}$$

$$I = I_0 \sin (\omega t - \theta)$$

where θ is the phase difference between current and voltage.

Hence the power in an A.C. circuit at any instant = $E \cdot I$

$$= E_0 \sin \omega t \times I_0 \sin (\omega t - \theta)$$

$$= E_0 I_0 \sin \omega t (\sin \omega t \cos \theta - \cos \omega t \sin \theta)$$

$$= E_0 I_0 [\sin \omega t \cos \theta - \sin \omega t \cos \omega t \sin \theta]$$

$$= E_0 I_0 \left[\frac{1}{2} (1 - \cos 2\omega t) \cos \theta - \frac{1}{2} \sin 2\omega t \sin \theta \right]$$

$$= \frac{E_0 I_0}{2} [\cos \theta - \cos 2\omega t \cos \theta - \sin 2\omega t \sin \theta]$$

$$= \frac{E_0 I_0}{2} [\cos \theta - \cos (2\omega t - \theta)]$$

This shows that the power consumed also varies with time - Hence the average power P during each complete cycle is given by

$$\begin{aligned}
 P &= I/T \left[\int_0^T EI dt \right] \\
 &= I/T \cdot I/2 E_0 I_0 \int_0^T [\cos \theta - \cos(2\omega t - \theta)] dt \\
 &= \frac{E_0 I_0}{2T} \left[\int_0^T \cos \theta dt - \int_0^T \cos(2\omega t - \theta) dt \right] \\
 &= \frac{E_0 I_0}{2T} \left[\cos \theta \cdot t - \frac{\sin(2\omega t - \theta)}{2\omega} \right]_0^T \\
 &= \frac{E_0 I_0}{2T} \left[\cos \theta \cdot T - 0 - \frac{\sin(2\omega T - \theta)}{2\omega} + \frac{\sin(-\theta)}{2\omega} \right]
 \end{aligned}$$

$$\therefore T = 2\pi/\omega$$

$$\begin{aligned}
 \therefore P &= \frac{E_0 I_0}{2 \times 2\pi/\omega} \left[\cos \theta \frac{2\pi}{\omega} - \frac{\sin(4\pi - \theta)}{2\omega} + \frac{\sin(-\theta)}{2\omega} \right] \\
 &= \frac{E_0 I_0}{2} \cos \theta \left[\because \sin(4\pi - \theta) = \sin(-\theta) \right] \\
 &= \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos \theta
 \end{aligned}$$

$$\begin{aligned}
 \therefore P &= E_{r.m.s} \times I_{r.m.s} \times \cos \theta \\
 &= (\text{Virtual Volts}) \times (\text{Virtual amperes}) \times \cos \theta
 \end{aligned}$$

$$\therefore \text{Average Power} = \text{Virtual Power} \times \cos \theta$$

As $\cos \theta$ is the factor by which the product of the r.m.s. values of the voltage and current must be multiplied to give the power dissipated, it is known as the power factor of the circuit, when e.m.f. is expressed in the volts and current in amperes then the power is expressed in the watts and current in amperes then the power is expressed in watts.

Wattless Current

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The current A.C. Circuit is said to be wattless when the average power consumed in the circuit is zero. This is possible when the power factor is zero i.e. when $\cos \theta = 0$

$$\text{or } \theta = \pi/2$$

i.e. when the current and voltage differ in phase by 90° . For example, in a pure inductance the current lags behind the voltage by $\pi/2$. Hence the average power absorbed in a pure inductance is zero. This is explained by the fact that power is absorbed in the magnetic field of the coil during the first quarter cycle, but is returned back to the generator during the next quarter cycle. In a similar way the average power absorbed in a pure capacitance is also zero.